Introduction to hyperbolic conservation laws and their numerical solution (Prof. M. Putti, University of Padova – frontal lessons, 30 h; laboratory classes, 10 h)

Theory

- *Characteristic curves and classification of PDEs.*
  Well-posedness and the Cauchy problem.
- *Hyperbolic equations in one spatial dimension.*
  The Cauchy problem for the first order scalar equation; the one-dimensional wave equation; the inviscid Burgers equation. The method of characteristics; boundary conditions. Systems of first order equations; solutions obtained by gluing solutions to Riemann problems. Discontinuous solutions; propagation of singularities, necessity of defining generalized (weak) solutions for well-posedness.
- *Systems of conservation laws.*
  Conservation (divergence) form of the equations; Rankine-Hugoniot conditions; Lax Entropy conditions; Total Variation and Bounded Variation condition; vanishing viscosity solutions. Shock, rarefactions, contact discontinuities; wave front tracking.
- *Review of Numerical discretizations.*
  Finite difference methods: Lax, Friedrichs, Wendroff methods; Godunov method and the Finite Volume method. Numerical fluxes and the solution of the Riemann problem. Extension to higher order approximations; Extension to multidimensional problems. Numerical solution of the system of ODEs; stability and stiffness for implicit and explicit methods. Treatment of source terms.
- *Examples of applications.*
  Multiphase flow in porous media. Shallow Water Equations.

Laboratory activity

The students will be required to work under the supervision of the instructor on model problems from the theoretical and numerical point of view. At the end, each group will prepare a lecture discussing the theory and the particular numerical implementation adopted, highlighting numerical results and open questions.

Minimum skills

The course discusses issues related to the theory of hyperbolic equations and systems of nonlinear conservation laws. The requested mathematical background is the classical education of Italian engineering undergraduate schools, including standard multidimensional calculus and notions of linear algebra and numerical analysis. The course maintains a practical flavor, discussing a number of examples and applications, and for this reason it sacrifices mathematical rigor replacing formal statements with intuitive explanations often based on physical principles.

Final exam

The exam is based on the presentation and discussion of the laboratory activity.

Suggested references

Course notes prepared by the teacher will be available to all participants.