UNIVERSITÀ DEGLI STUDI DI PADOVA

PHD COURSES

Course unit English denomination	Numerical Methods
SS	MATH-05/A
Teacher in charge	Luca Bergamaschi Andrea Franceschini
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	January 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☒ Yes (75% minimum of presence) ☐ No
Course unit contents	Part 1: Iterative methods for large linear and nonlinear systems. Sparse matrices. Preliminaries on iterative methods. The method of the steepest descent. The Conjugate Gradient method. Convergence theory. Acceleration of iterative methods by preconditioning. Krylov subspace methods. The GMRES method. Practical implementations. Iterative solution of large systems of nonlinear equations: The Newton method and its variants. Local convergence, hints to global convergence. Inexact Newton methods. Quasi-Newton methods. Part 2: Introduction to Finite Elements for elliptic and parabolic equations. Second order partial differential equations (PDEs): elliptic, parabolic, and hyperbolic equations. Boundary and initial conditions. Variational methods: Galerkin methods and weak formulations. Time integration for parabolic PDEs. Finite elements: 1D Lagrangian elements, extensions to 2D and 3D, triangular finite elements. Finite element solution of Poisson's equation and diffusion equation.
Learning goals	Knowledge of Iterative Methods: Understand the main concepts of iterative methods for solving large systems of linear and nonlinear equations. Apply the steepest descent method and the conjugate gradient method as basic techniques for solving linear systems. Analysis of Convergence: Develop a thorough understanding of the convergence theory of iterative methods. Assess the effectiveness of acceleration methods through preconditioning to improve convergence. Use of Krylov Methods: Apply methods based on Krylov subspaces, including the GMRES method, for the solution of large systems. Implement and solve practical problems using these iterative methods. Nonlinear Systems: Address the solution of large systems of nonlinear equations using Newton's method and its variants. Understand the concepts of local convergence and global convergence in nonlinear contexts. Apply quasi-Newton methods to efficiently solve large nonlinear systems.

	Introduction to Finite Elements: Study the characteristics of second- order PDEs, namely elliptic, parabolic, and hyperbolic equations, with special attention to boundary and initial conditions. Variational Methods: Be familiar with variational methods, particularly the Galerkin method and weak formulations for solving PDE problems. Develop skills in the analysis and implementation of time integration methodologies for parabolic PDEs. Finite Elements in Practice: Design and implement finite elements, with particular attention to Lagrangian elements in 1D and their extensions in 2D and 3D, including triangular finite elements. Solve practical applications of the Poisson equation and diffusion equation using finite element techniques.
Teaching methods	Lecture with support of projected slides.
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Basic concepts of linear algebra and mathematical analysis. Attendance of the course “Elements of tensor and numerical algebra”.
Examination methods (in applicable)	Delivery of a report that describes the solution to an exercise provided by the teacher.
Suggested readings	Slides provided during the lectures and these textbooks: 1. Y. Saad: Iterative methods for sparse linear systems, SIAM, 2003 2. C.T. Kelley. Iterative methods for linear and nonlinear equations, SIAM, 1987 3. A. Quarteroni: Numerical models for differential problems, Springer (2014). 4. O. C. Zienkiewicz, R. L. Taylor, J. Z. Zhu: The finite element method: its basis and fundamentals, Butterworth-Heinemann 2005).
Additional information	None