UNIVERSITÀ DEGLI STUDI DI PADOVA

PHD COURSES

Course unit English denomination	Elements of tensor and numerical algebra
Teacher in charge (if defined)	Giovanna Xotta (course instructor), Matteo Frigo
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	December
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☒ Yes (70% minimum of presence) ☐ No
Course unit contents	Vector and tensor algebra: Algebra of vectors: index notation; addition and multiplication by a scalar; dot and cross product. Algebra of second-order tensors: matrix notation; addition and multiplication by scalars; dot and tensor product; transpose and inverse of a tensor; orthogonal tensor; symmetric and skew tensors; tensor invariants. Higher-order tensors. Transformation laws for basis vectors and components: vectorial and tensorial transformation laws; isotropic tensors. General bases: general basis vectors; covariant and contravariant components of a vector; covariant, contravariant and mixed components of a tensor. Tensor analysis: Gradient and divergence operators: gradient of a scalar field; concept of directional derivative; gradient and divergence of a vector field and of a second-order tensor; Laplacian and Hessian. Integral theorems: Divergence theorem; Stokes' theorem. Numerical linear algebra: Square matrices and eigenvalues: norms, subspaces associated to a matrix, canonical forms. Orthogonal vectors: Gram-Schmidt and Householder recurrences. Types of matrices: normal and Hermitian matrices, nonnegative matrices, M-matrices, positive definite matrices. Projection operators: range and null spaces, matrix representation, orthogonal projections. Elements of functional analysis: Preliminaries: definitions, norms, inner product, Hölder inequality. Types of spaces: Banach, Hilbert and Sobolev spaces, square-integrable functions, Lp spaces. Variational formulation: functionals, Euler-Lagrange equations, weak formulation, Green's lemma, forms.
Learning goals	This course is designed to offer a solid foundation in topics essential for various specialized doctoral courses. The first part focuses on the key concepts of tensor algebra, which are frequently encountered in many research books and articles. Through these topics, students will acquire the theoretical and practical tools necessary for advanced academic research. In particular, they will develop skills in vector

	and tensor algebra, mastering key operations and transformations, as well as tensor analysis techniques such as gradients, divergences, and integral theorems. The second part introduces the basic principles of numerical linear algebra, covering matrix and vector theory, which are fundamental for the computer implementation of mathematical models. Specifically, students will explore square matrices, eigenvalues, norms, canonical forms, orthogonalization techniques, matrix classifications, and projection operators, while also introducing essential elements of functional analysis, such as function spaces and variational formulations.
Teaching methods	Frontal lesson
Course on transversal, interdisciplinary, transdisciplinary skills	☒ Yes ☐ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	/
Examination methods (in applicable)	Written exam, to assess the adequacy and completeness of the knowledge acquired.
Suggested readings	1. Lecture notes 2. Suggested books: - J. Bonet, R.D. Wood: Nonlinear Continuum Mechanics for Finite Element Analysis, Cambridge university press, 2008. - G.A. Holzapfel: Non linear solid mechanics: A continuum approach for engineering, John Wiley and Sons, 2000. - A. Quarteroni: Numerical models for differential problems, Springer, 2014. - Y. Saad: Iterative methods for sparse linear systems, SIAM, 2003.
Additional information	/