ELEMENTS OF TENSOR AND NUMERICAL ALGEBRA

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Program:

Vector and tensor algebra:

Algebra of vectors: index notation; addition and multiplication by a scalar; dot and cross product. Algebra of second-order tensors: matrix notation; addition and multiplication by scalars; dot and tensor product; transpose and inverse of a tensor; orthogonal tensor; symmetric and skew tensors; tensor invariants. Higher-order tensors. Transformation laws for basis vectors and components: vectorial and tensorial transformation laws; isotropic tensors. General bases: general basis vectors; covariant and contravariant components of a vector; covariant, contravariant and mixed components of a tensor.

Tensor analysis: Gradient and divergence operators: gradient of a scalar field; concept of directional derivative; gradient and divergence of a vector field and of a second-order tensor; Laplacian and Hessian. Integral theorems: Divergence theorem; Stokes' theorem.

Numerical linear algebra:

Square matrices and eigenvalues: norms, subspaces associated to a matrix, canonical forms. Orthogonal vectors: Gram-Schmidt and Householder recurrences. Types of matrices: normal and Hermitian matrices, nonnegative matrices, M-matrices, positive definite matrices. Projection operators: range and null spaces, matrix representation, orthogonal projections.

Elements of functional analysis:

Preliminaries: definitions, norms, inner product, Holder inequality. Types of spaces: Banach, Hilbert and Sobolev spaces, square-integrable functions, L^p spaces. Variational formulation: functionals, Euler-Lagrange equations, weak formulation, Green's lemma, forms.

References:

1. J. Bonet, R.D. Wood: *Nonlinear Continuum Mechanics for Finite Element Analysis*, Cambridge university press, 2008.
2. G.A. Holzapfel: *Non linear solid mechanics: A continuum approach for engineering*, John Wiley and Sons, 2000.
3. A. Quarteroni: *Numerical models for differential problems*, Springer, 2014.
4. Y. Saad: *Iterative methods for sparse linear systems*, SIAM, 2003.

Examination and grading:

Final evaluation will be based on a written test, to verify the adequacy and completeness of the knowledge acquired.

Course details:

The course will be held in person. Details on dates and location will be available at: https://www.dicea.unipd.it/en/phd-course/calendar