Introduction to Partial Differential Equations for the applied sciences (Prof. N. Garofalo, University of Padova - frontal lessons, 30 h)

Lesson 1: Preliminaries on integration. Fourier transform on the line. Solution of the initial value problem for the 1-dimensional homogeneous transport equation.
Lesson 2: Solving the non-homogeneous 1-dimensional transport equation. Convolution.
Lesson 3: Solving the Cauchy problem for the 1-dimensional wave equation. The method of variation of parameters.
Lesson 4: Differentiability: basic properties and results. The Laplace operator. Sub- and super-harmonic functions. Gravitational and electrostatic potentials.
Lesson 5: Fourier transform in R^n. The Fourier transform of the surface measure on a sphere (the role of Bessel functions). Solution of the homogeneous Cauchy problem for the three-dimensional wave equation.
Lesson 6: Bonaventura Cavalieri's principle. Computation of the area and volume of the (n-1)-dimensional unit sphere and the omnipresent Gaussian and gamma functions. Kirchoff formula and Huygens principle.
Lesson 8: Solution of the non-homogeneous Cauchy problem for the wave equation in the physical space (part 2). Delayed potential. Uniqueness of the physical solution (part 1). Monotonicity of the energy in the light cone.
Lesson 9: End of the proof of uniqueness. The heat equation. Fourier solution of the homogeneous heat equation and its physical limitations.
Lesson 10: The Fourier transform of a Gaussian. Fundamental solution of the heat equation. Behavior as time tends to zero of the solution and consistency with the initial conditions.
Lesson 11: Non-uniqueness for the heat equation (the growth condition of Tikhonov). Stationary solutions of the heat equation. The return of harmonic functions.
Lesson 12: Weak maximum principle. The Dirichlet problem and its solvability in some basic situations. Invariance of Laplace operator under orthogonal transformations. Spherically symmetric solutions of the Laplacian.
Lesson 13: Hessian and Laplacian of a spherically symmetric function. Solving the Dirichlet problem in a ball via the maximum principle and symmetry considerations. From a PDE to an ODE.
Lesson 14: Mean value inequalities for sub- and super-harmonic functions. The strong maximum principle. The St. Venant principle and the torsion function of a homogeneous beam fixed at one of its ends.
Lesson 15: An identity of Rellich. A symmetry result in potential theory: why in industry it is best to build beams with spherical cross section.

Suggested reference:
Nicola Garofalo, "*Introductory PDE's for the applied sciences*", Lecture Notes, Libreria Progetto, Padova.
Additional references will be suggested during the lectures